Plasma profiles and their control

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Outline I

- Profile control
- 2 Tokamak profile dynamics Magnetic field diffusion Kinetic transport Coupling to 2D equilibrium
- 3 Profile control Actuators Sensors
 - Control

Section 1

Profile control

Introduction

- Plasma held place by TF+PF coils, magnetic equilibrium described by Grad-Shafranov equation. To determine equilibrium we need to know the internal profiles $p'(\psi)$, $T(\psi) = RB_{\phi}$.
- But what determines this internal pressure and poloidal current distribution?

Introduction

- Plasma held place by TF+PF coils, magnetic equilibrium described by Grad-Shafranov equation. To determine equilibrium we need to know the internal profiles $p'(\psi)$, $T(\psi) = RB_{\phi}$.
- But what determines this internal pressure and poloidal current distribution?
- We will discuss internal evolution (=transport) of current, particles and energy.
 - 1D problem: transport || B is ∞-fast, only transport ⊥ B evolves slowly. (Q:How slowly?)
 - Plasma profiles: important for stability and performance.
 - We will (roughly) derive 1D transport equations for *j*, *T*, *n*.
- What are the actuators? How do they affect profile evolution?

Section 2

Tokamak profile dynamics

Subsection 1

Magnetic field diffusion

B field diffusion in resistive medium

Ohm's law for resistive MHD:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{j} \tag{1}$$

assume $\mathbf{v} = \mathbf{0}, \, \eta = \mathbf{cst}$ and use $\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$

$$\mathbf{E} = \frac{\eta}{\mu_0} (\nabla \times \mathbf{B}) \tag{2}$$

Take $\nabla \times$ both sides and use $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{\eta}{\mu_0} \nabla \times (\nabla \times \mathbf{B}) = \frac{\eta}{\mu_0} \nabla^2 \mathbf{B}$$
 (3)

Typical time scales

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{\eta}{\mu_0} \nabla^2 \mathbf{B} \tag{4}$$

- Superconducting case: $\eta=0 o rac{\partial \mathbf{B}}{\partial t}=0$ (Field frozen in medium)
- Conducting case, typical time scales depend on resistivity and system size $\tau = \frac{\mu_0 L^2}{n}$

Time scales w.r.t. shot time

 On what time scale does the plasma B field transport evolve with respect to the shot time?

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- Exercise:
 - Calculate typical magnetic diffusion times for TCV, JET and ITER and compare to typical plasma shot times. Recall $\tau=\frac{\mu_0L^2}{\eta}$ and use

$$\eta pprox 1.8 imes 10^{-2} T_e^{-3/2} [{
m eV}]$$

- TCV: L = 0.25 m, $T_e = 1 \text{keV}$, $t_{shot} = 2 \text{s}$
- JET: L = 1 m, $T_e = 3 \text{keV}$, $t_{shot} = 12 \text{s}$
- ITER: L = 2 m, $T_e = 5 \text{keV}$, $t_{shot} = 300 \text{s}$

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Answer:

- TCV: 0.14s
- JET: 11.4s
- ITER: 100s

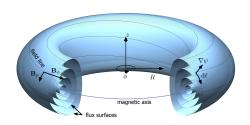
NB: Slow internal dissipation of **B** field can not be ignored.

- Sketch of derivation. Full derivations in [1], [2], [3]
- Write Ohm's law (without flows) with conductivity $\sigma = 1/\eta$

$$\underbrace{\mathbf{j}}_{\text{total current}} = \underbrace{\sigma \mathbf{E}}_{\text{inductive/Ohmic current}} + \underbrace{\mathbf{j}}_{\text{ni}}$$
 total current inductive/Ohmic current non-inductive current (5

 Project in direction || B and average over a flux surface (from Grad-Shafranov equilibrium)

$$\frac{\langle \mathbf{j} \cdot \mathbf{B} \rangle}{B_0} = \frac{\sigma_{\parallel} \langle \mathbf{E} \cdot \mathbf{B} \rangle}{B_0} + \frac{\langle \mathbf{j}_{ni} \cdot \mathbf{B} \rangle}{B_0}$$
 (6)



- $\psi(R, Z) = \int \mathbf{B} \cdot d\mathbf{A}_Z$, loci of constant ψ define flux surfaces.
- Define toroidal flux through a flux surface:

$$\Phi = \int \mathbf{B} \cdot d\mathbf{A}_{\phi} = \int B_{\phi} dA_{\phi}, \tag{7}$$

associate radial metric $\rho = \sqrt{\Phi}$ and $\rho_N = \rho/\rho_b$ (b = boundary)

Exercise:

• Show that in the case of circular cross section, large aspect ratio $(R/a \to \infty)$ and low plasma current $(B_\phi \approx B_0)$ it holds that $\rho_N = r/a$, where r is the geometric radius and a is the radius of the LCFS.

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Solution:

• $B_{\phi}=B_0$, circular cross section, then $\Phi=\pi r^2 B_0$ and $\rho=r\sqrt{\pi B_0}$, $\rho_{edge}=a\sqrt{\pi B_0}$ so $\rho_N=r/a$

Flux Diffusion: Sketch of derivation

 Using vector calculus, differential calculus, plus Maxwell's equations, we can write the equation as a function of ρ:

$$\underbrace{\frac{\langle \mathbf{j} \cdot \mathbf{B} \rangle}{B_0}}_{= \underbrace{\frac{\sigma_{\parallel} \langle \mathbf{E} \cdot \mathbf{B} \rangle}{B_0}}_{\sim \sigma_{\parallel} \underbrace{\frac{\partial \psi(\rho, t)}{\partial t}}} + \underbrace{\frac{\langle \mathbf{j}_{ni} \cdot \mathbf{B} \rangle}{B_0}}_{j_{ni}(\rho)}$$
(8)

Flux surface averaging

Volume:

$$V = \int dV = \int R d\phi \frac{d\psi}{|\nabla \psi|} d\ell_{p} = \int \left(\oint \frac{d\ell_{p}}{B_{p}} \right) d\psi$$
 (9)

$$\frac{\partial V}{\partial \psi} = \oint \frac{\mathrm{d}\ell_{p}}{B_{p}} \tag{10}$$

Flux Surface average of Q:

$$\langle Q \rangle \equiv \frac{\partial}{\partial V} \int Q dV = \frac{\partial \psi}{\partial V} \frac{\partial}{\partial \psi} \oint Q \frac{R d\ell}{|\nabla \psi|} d\psi d\phi$$

$$= \oint Q \frac{d\ell_p}{B_p} / \oint \frac{d\ell_p}{B_p}$$
(11)

Flux Diffusion: Sketch of derivation

$$\sigma_{\parallel} \frac{\partial \psi}{\partial t} = \frac{F^2}{(2\pi)^4 \mu_0 B_0^2 \rho} \frac{\partial}{\partial \rho} \left(\frac{g_2 g_3}{\rho} \frac{\partial \psi}{\partial \rho} \right) - \frac{V'}{2\pi \rho} j_{ni}$$
(12)

Where:

- $g_2 = \left\langle \frac{|\nabla V|^2}{R^2} \right\rangle$, $g_3 = \left\langle 1/R^2 \right\rangle$, $V' = dV/d\rho$. (Geometric profiles)
- $F = RB_{\phi} = R_0B_0 + \text{small correction due to poloidal currents.}$
- σ_{\parallel} is the conductivity (roughly $\sim T_e^{3/2}$)
- j_{ni} is non-inductive current drive (self-generated, or from auxiliary sources)

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Recall: (Ohmic \mathbf{j}) = (Total \mathbf{j}) - (non-inductive \mathbf{j}) Special cases:

- $j_{ni}=0$: "Ohmic" plasma: all current sustained by inductive part (LHS). Requires $\frac{d\psi}{dt}>0 \ \forall \ t$.
- $\frac{d\psi}{dt} = \text{constant: } stationary \ state. \ \frac{\partial}{\partial \rho} \frac{\partial \psi}{\partial t} = \frac{\partial}{\partial t} \frac{\partial \psi}{\partial \rho} = 0. \ \text{Fixed plasma}$ current profile (shape) but not steady-state.
- $\frac{d\psi}{dt} = 0$: fully *steady-state* plasma. Plasma current sustained entirely by j_{ni} **desired for reactor**

Non-inductive current drive

Sources of non-inductive current drive: $j_{ni} = j_{bs} + j_{aux}$

- Bootstrap current: self-generated plasma current due to trapped particle orbits. ~ ∇p. Equations in [4], [5]
- • Auxiliary current: driven by auxiliary systems: NBI, EC, LH, IC \rightarrow NBCD, ECCD, LHCD, ICCD.

In a reactor, one must maximize the bootstrap current fraction ($\sim 70\%$) since auxiliary current drive has large a capital cost and lowers Q_{fus} . Most plasma current becomes self-generated.

Boundary conditions for poloidal flux

Boundary conditions:

Impose total plasma current

$$\left[\frac{G_2}{\mu_0}\frac{\partial\psi}{\partial\rho}\right]_{\rho=\rho_0}=I_p(t). \tag{14}$$

• But, physically, $\psi(\rho_b)$ is imposed by 2D equilibrium...

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Interpretation:

• To have nonzero I_p we need $\left[\frac{G_2}{\mu_0}\frac{\partial\psi}{\partial\rho}\right]_{\rho=\rho_e}\neq 0$, but $\psi(\rho)$ evolution is governed by diffusion equation that evolves to $\frac{\partial\psi}{\partial\rho}=0$. How do we maintain I_p ?

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- Two possibilities
 - Time-varying $\psi(\rho_b)$ (induced current)
 - Use source term (non-inductive current)

Other measures of current distribution

 In practice, physicists like to work with q profile since it is indicative of MHD stability.

$$q(\psi) = \frac{1}{2\pi} \oint \frac{1}{R} \frac{B_{\phi}}{B_{p}} d\ell = \frac{T(\psi)}{2\pi} \oint \frac{d\ell}{R^{2}B_{p}}$$
(15)

It can also be shown that $q \equiv \frac{\partial \Phi}{\partial \psi}$.

• Plasma stability, performance, transport is influenced by q and its spatial derivative the *magnetic shear* $s = \frac{\rho}{q} \frac{\partial q}{\partial \rho}$.

Question: response to step in auxiliary current drive

$$\sigma_{\parallel} \frac{\partial \psi}{\partial t} = \frac{F^2}{(2\pi)^4 \mu_0 B_0^2 \rho} \frac{\partial}{\partial \rho} \left(\frac{g_2 g_3}{\rho} \frac{\partial \psi}{\partial \rho} \right) - \frac{V'}{2\pi \rho} j_{ni}$$
(16)

- Question:
 - Suppose we are in a stationary state with $\frac{\partial \psi}{\partial t} = c$.
 - Qualitatively, what happens if we instantaneously increase j_{ni}?
 - · Does the current density profile change instantaneously?
 - Which physical law does this example represent?

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Answer:

- We can not change the 1st term on the LHS since its time evolution is governed by (16) itself! Thus, instantaneously, only the ohmic current density $\sim \frac{\partial \psi}{\partial t}$ changes.
- This is an example of Lenz's law: an inductive circuit resists a change in flux. Also called "back-EMF".

Subsection 2

Kinetic transport

Thermal energy transport equation

• Diffusive transport law for energy density $W = k_B T_e n_e$

$$\frac{\partial W}{\partial t} + \frac{1}{\rho} \frac{\partial}{\partial \rho} \Gamma = \text{Sources}$$
 (17)

with diffusive flux $\Gamma = \rho D \frac{\partial W}{\partial \rho}$ with diffusion coefficient D.

In toroidal geometry, this is more complicated. For each species:

$$\frac{3}{2}(V')^{-5/3}\left(\frac{\partial}{\partial t}\left[(V')^{5/3}n_{\alpha}T_{\alpha}\right]\right) + \frac{1}{V'}\frac{\partial}{\partial \rho}\left(q_{\alpha} + \frac{5}{2}T_{\alpha}\Gamma_{\alpha}\right) = P_{\alpha}$$
(18)

- $V' = \frac{\partial V}{\partial \rho}$ (geometry)
- · Similarly for particle transport
- For each species (e⁻, i⁺, impurities)!

Transport coefficients

- Diffusive/advective fluxes in plasma depend in a complicated way on the plasma itself, nonlinear multi-scale turbulence.
- For modeling/control purposes, must resort to simple models.
- Recent breakthrough: Neural Network emulation of Gyrokinetic fluxes from the QuaLiKiz quasilinear gyrokinetic code [6], [7].
 Speedup from 1hour to 1ms per time step.

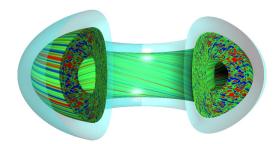


Figure: Gyrokinetic simulations of tokamak turbulence. Source: GYRO/PPPL

Models for diffusive and convective flux

• Transport equation, for species α :

$$\frac{3}{2}(V')^{-5/3}\left(\frac{\partial}{\partial t}\left[(V')^{5/3}n_{\alpha}T_{\alpha}\right]\right) + \frac{1}{V'}\frac{\partial}{\partial \rho}\left(q_{\alpha} + \frac{5}{2}T_{\alpha}\Gamma_{\alpha}\right) = P_{\alpha}$$
(19)

where q_{α} is the diffusive flux and Γ_{α} is the convective flux.

$$\frac{q_{\alpha}}{n_{\alpha}T_{\alpha}} = -V'G_{1}\langle(\nabla\psi)^{2}\rangle \sum_{\beta \in \text{all species}} \left(\chi_{\alpha\beta}^{T} \frac{1}{T_{\beta}} \frac{\partial T_{\beta}}{\partial \rho} + \chi_{\alpha\beta}^{n} \frac{1}{n_{\beta}} \frac{\partial n_{\beta}}{\partial \rho}\right)$$
(20)

$$\frac{\Gamma_{\alpha}}{n_{\alpha}} = -V'G_1\langle (\nabla \psi)^2 \rangle \sum_{\beta \in \text{all species}} \left(D_{\alpha\beta}^T \frac{1}{T_{\beta}} \frac{\partial T_{\beta}}{\partial \rho} + D_{\alpha\beta}^n \frac{1}{n_{\beta}} \frac{\partial n_{\beta}}{\partial \rho} \right)$$
(21)

Qualitative picture of transport coefficients

- Accurate and simple models for thermal transport do not exist (yet). We can only make some qualitative statements.
- Usually diagonal terms are dominant, e.g. χ_{ee} for electrons.
- Usually diffusive terms are assumed dominant. $D_{\alpha\beta}=0$
- Then T_e diffusion equation becomes, e.g.

$$\frac{3}{2}(V')^{-5/3}\left(\frac{\partial}{\partial t}\left[(V')^{5/3}n_{e}T_{e}\right]\right) = \frac{1}{V'}\frac{\partial}{\partial \rho}\left(V'G_{1}\chi_{ee}n_{e}\frac{\partial T_{e}}{\partial \rho}\right) + P_{e}$$
(22)

Qualitative picture of transport coefficients

- Higher plasma current gives more confinement.
- Transport is 'stiff'. Above a critical $\frac{\nabla T_e}{T_e}$, χ_e increases drastically. Increasing $\frac{\nabla T_e}{T_e}$ above this threshold requires much more power (limited by actuators).
- High magnetic shear and low q are 'good' (low transport). Volume-averaged $\langle s/q \rangle$ [8]
- Shear close to 0 or negative is also good, can give 'internal transport barriers' (ITBs). e.g. [9], [10]

Sources and sinks for thermal energy

- Sources
 - Ohmic heating power (resistivity) $\sim \langle \mathbf{j} \cdot \mathbf{E} \rangle$
 - Local heat deposition by auxiliary systems
 - Collisional energy exchange with other species (α particles!)
- Sinks
 - Radiation losses (line radiation, cyclotron radiation, Bremsstrahlung)
 - · Conduction (loss to the wall)
 - Collisional energy exchange with other species.

Subsection 3

Coupling to 2D equilibrium

Coupling to 2D equilibrium

In reality, 1D transport is tightly coupled to 2D equilibrium (Grad-Shafranov). Geometric factors g_2 , g_3 come from 2D $\psi(R,Z)$ distribution

- Given a pressure and current distribution, GS equation determines 2D equilibrium. Very fast (Alfven = μs) timescales
- Given a 2D equilibrium geometry, 1D transport equations evolve distribution of current and pressure. Confinement time scales vs current redistribution time scales. ITER: $\tau_F \sim 5 \text{s}$, $\tau_{crt} \sim 100 \text{s}$.

In practice: very complicated, strongly nonlinear coupling between two sets of equations. Few good simulators exist.

Section 3

Profile control

Subsection 1

Actuators

Actuators

- Poloidal flux (current distribution)
 - I_p (boundary)
 - σ_{\parallel} or j_{BS} (heating changes resistivity or BS current)
 - j_{aux} (direct source).
 - Shape changes.
- Thermal transport:
 - Heating (direct source)
 - Change of transport via changing turbulence $q, s, \nabla T_e, \nabla n_e, \dots$
- Particle transport
 - · Gas (edge)
 - Neutral beam (internal source)
 - · Change particle transport.

Subsection 2

Sensors

Sensors (diagnostics) for profile control

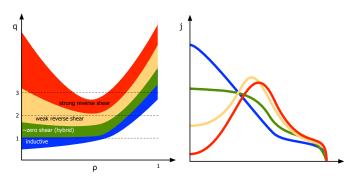
- Kinetic profiles: ECE, Thomson, interferometry, ...
- Magnetic profiles: MSE, Polarimetry, ...
- 2D equilibrium: RT equilibrium reconstruction, ...
- Different sampling rates, varying radial location of measurements, technically challenging diagnostics.
- How to merge information from various sources? State observers (Previous lecture)

Subsection 3

Control

Control Objectives

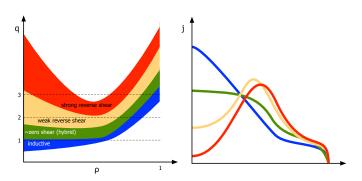
• Obtain plasma with desired profile shapes ("scenario"), possibly in stationary condition, at end of current ramp-up.



• Q: why is the inductive current profile peaked?

Control Objectives

• Obtain plasma with desired profile shapes ("scenario"), possibly in stationary condition, at end of current ramp-up.



- Q: why is the inductive current profile peaked?
- A: $\sigma_{neo} \sim T_e^{3/2}$

Control strategies

- Open-loop vs closed-loop control
 - Open-loop: design actuator time trajectories to get desired plasma state. (everyday practice for tokamak operators who program plasma discharges)
 - Closed-loop: attempt to track a reference: reject disturbances, robust against model uncertainties, etc (done very rarely - desired in the future)
- Both can benefit from model-based design tools

Model-based open-loop trajectory design

- Solve open-loop trajectory optimization problem.
- Include cost function (what you want) and constraints (what is possible)
- Simulation examples in literature: [11] [12], [13], to be tested in practice.

Examples of closed-loop profile control

- Non model-based
 - Control q_0 and q_{min} in DIII-D ramp-up using NBI [14]
 - Plasma regime control by LH power in Tore Supra [Imbeaux EPS 2009]
- Model-based: system identification of linear model arond operating point
 - Linear optimal feedback control around set point (JET, DIII-D) [15], [16].
- · Model-based: first principle models
 - Model-based controllers tested on DIII-D [17], [18], [?]
 - · Lyapunov-based controller design [19]
 - · Adaptive model based on linearization [20]
 - Model Predictive Control [21], [22]

Bibliography I



Hinton, F.L. et al. 1976 Rev. Mod. Phys. 48 239



Pereverzev, G.V. et al. 2002 {ASTRA} {A}utomated {S}ystem for {TR}ansport {A}nalysis in a {T}okamak Technical Report 5/98 IPP Report



Felici, F. 2011 Real-Time Control of Tokamak Plasmas: from Control of Physics to Physics-Based Control Ph.D. thesis Ecole Polytechnique Fédérale de Lausanne EPFL Lausanne



Sauter, O. et al. 1999 Physics of Plasmas 6 2834



Sauter, O. et al. 2002 Plasma Physics and Controlled Fusion 44 1999



Felici, F. et al. 2018 Nuclear Fusion 58 096006



van de Plassche, K.L. et al. 2020 Physics of Plasmas 27 022310



Citrin, J. et al. 2012 Plasma Physics and Controlled Fusion 54 65008



Connor, J.W. et al. 2004 Nuclear Fusion 44 R1



Goodman, T.P. et al. 2005 Plasma Physics and Controlled Fusion 47 B107



Felici, F. et al. 2012 Plasma Physics and Controlled Fusion 54 025002

Bibliography II





Ferron, J.R. et al. 2006 Nuclear Fusion 46 L13

Moreau, D. et al. 2008 Nuclear Fusion 48 106001

Moreau, D. et al. 2011 Nuclear Fusion **51** 63009

Ou, Y. et al. 2008 Plasma Physics and Controlled Fusion 50 115001 (24pp)

Barton, J.E. et al. 2012 Nuclear Fusion **52** 123018

Argomedo, F.B. et al. 2013 Nuclear Fusion 53 033005

Kim. S. et al. 2012 Nuclear Fusion **52** 074002

Maljaars, E. et al. 2015 Nuclear Fusion 55 023001

Maljaars, E. et al. 2017 Nuclear Fusion 57 126063